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DISPERSE SYSTEMS WITH SUSPENDED PARTICLES: THE PROBLEM OF SCALING AND HYDRODYNAMIC-SIMILARITY NUMBERS

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Within the framework of similarity theory, a rather universal method is developed for generalizing experimental data on the hydrodynamics of a wide class of disperse systems with ascending motion of a gas. This method is based on use of the excess gas velocity $u - u_t^*$, which is a measure of the kinetic energy of the particles. Using this quantity, the hydrodynamic-similarity numbers Fr_t^* and $\bar{J}_s^*$ are formulated, which are generalized characteristics of such systems. Specific examples of applying this method are given.

In practice, wide use is made of disperse systems with suspended particles. These are a fluidized bed, vertical pneumatic transport, and an intermediate disperse medium, i.e., a circulating fluidized bed. The study of various aspects of the behavior of these systems is the concern of extensive literature, for example, [1-6]. The methods developed for description, as a rule, are highly specialized, i.e., they are adapted to a specific individual system. In the present work we attempt to analyze in a unified context the problem of scaling in all three systems. As a result, we determine the minimum number of dimensionless groups that are composed of dimensional independent variables and that completely determine the similarity of hydrodynamic processes in the above-mentioned disperse systems. The possibility for this problem to be solved is based on the presence of one very substantial similarity in these systems: the weight of the particles in them is compensated by the friction force against the gas, and the entire excess power $\Delta p(u - u_t^*)$ expended by the fan goes to acceleration of the suspended particles and creation of the complex picture of their collective motion. The latter can include translational and pulsational motion, internal circulation, etc. The main difference between the disperse systems considered consists in the specific features of this motion:

1) in a fluidized bed overall directed motion of the solid phase is absent, while the presence of gas bubbles provides intense pulsational motion of the particles against a background of their internal circulation (in wakes of bubbles – upward, in the remaining emulsion phase – downward); here the existence of some circulation loops is possible [6];

2) a circulating fluidized bed is characterized by rather intense motion of the particles in a single internal circulation loop (in the bed core – upward, in an annular zone at the riser walls – downward) against a background of overall directed motion;

3) vertical pneumatic transport is upward-directed motion of the particles over the entire cross section of the riser against a background of small-scale pulsational motion.

In all the variations in the character of the particle motion in the systems considered the excess gas velocity is a measure of the intensity of this motion. It is apparent that the quantity $u - u_t^*$ can be a charactersitic scale of the velocity in describing the hydrodynamic processes within the framework of similarity theory. As determining linear scales of the collective particle motion, it is natural to take geometric dimensions of the system, namely, the height of the bed (riser) H and its diameter D. With account for these assumptions we can write the following equation for determining the desired hydrodynamic characteristic:

$$\Gamma = f\left(\begin{Bmatrix} J_s \\ H_0 \end{Bmatrix},\ \rho_s,\ u - u_t^*,\ g, h, H, D\right). \tag{1}$$

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It should be noted that the parameters J_s and H_0, which determine the mass of the disperse material in the system, are mutually exclusive: in accounting for the parameter H_0 (a fluidized bed; a circulating fluidized bed operating in accordance with the "furnace" scheme, when the mass of the material in the riser is assigned as $M = \rho_s(1 - \varepsilon_0)H_0S$), the parameter J_s is neglected in Eq. (1). And, conversely, in assigning J_s (a circulating fluidized bed operating by the "chemical reactor" scheme; vertical pneumatic transport) the parameter H_0 is neglected in Eq. (1).

Using the π-theorem of dimension theory [7], we write the dimensionless analog of (1)

$$\Gamma' = f\left(\begin{Bmatrix} \bar{J}_s^* \\ H_0/H \end{Bmatrix}, \mathrm{Fr}_t^*, \frac{h}{H}, \frac{H}{D}\right), \tag{2}$$

which with allowance for the aforesaid gives two equations:

a) for a fluidized bed and a circulating fluidized bed – a "furnace"

$$\Gamma' = \varphi\left(\frac{H_0}{H}, \mathrm{Fr}_t^*, \frac{h}{H}, \frac{H}{D}\right); \tag{3}$$

b) for a circulating fluidized bed – a "chemical reactor" – and vertical pneumatic transport

$$\Gamma' = \psi\left(\bar{J}_s^*, \mathrm{Fr}_t^*, \frac{h}{H}, \frac{H}{D}\right). \tag{4}$$

Obtained relations (3) and (4) contain a number of dimensionless governing parameters and allow one to establish rules of scaling for the hydrodynamic processes. These relations include two characteristic combinations $\mathrm{Fr}_t^* = (u - u_t^*)^2/gH$ and $\bar{J}_s^* = J_s/\rho_s(u - u_t^*)$ that have the following physical meaning: Fr_t^* is the ratio of the kinetic energy of the particles to their potential energy; $\bar{J}_s^*$ is the concentration of the particles in the system. Using specific examples, we will show that Eqs. (3) and (4) are actually a general form of dimensionless relations that generalize experimental data.

1) *A fluidized bed.* For the diameter of the gas bubbles the following equation was derived in [8]:

$$\frac{D_b}{h} = 1.3\left[\mathrm{Fr}\left(\frac{H}{h}\right)\right]^{1/3}, \tag{5}$$

which generalizes a large amount of experimental data (more than 20 works). The following formula for calculating the velocity of the gas bubbles was established in [9]:

$$\frac{V_b}{u - u_0} = 1.9\left[\mathrm{Fr}\left(\frac{H}{H_0}\right)\right]^{-1/3}\left(\frac{D}{H}\right)^{1/2}. \tag{6}$$

In [10], it was suggested that the expansion (concentration) of an inhomogeneous fluidized bed be calculated using the ratio

$$\frac{H - H_0}{H_0} = \frac{\varepsilon - \varepsilon_0}{1 - \varepsilon} = 0.7\left[\mathrm{Fr}\left(\frac{H}{H_0}\right)\right]^{1/3}\left(\frac{H_0}{D}\right)^{1/2}. \tag{7}$$

2) *A circulating fluidized bed.* This bed has properties of both a fluidized bed (in its lower portion) and a pneumatic-transport system (in the transport zone). For a bed operating by the "furnace" scheme, the following relation was obtained in [11] to calculate the distribution of the concentration ($\rho = \rho_s(1 - \varepsilon)$) of particles in the transport zone:

$$\frac{\rho}{\rho_s} = 0.053\,\mathrm{Fr}_t^{0.62}\left(\frac{h}{H}\right)^{-0.45}, \tag{8}$$

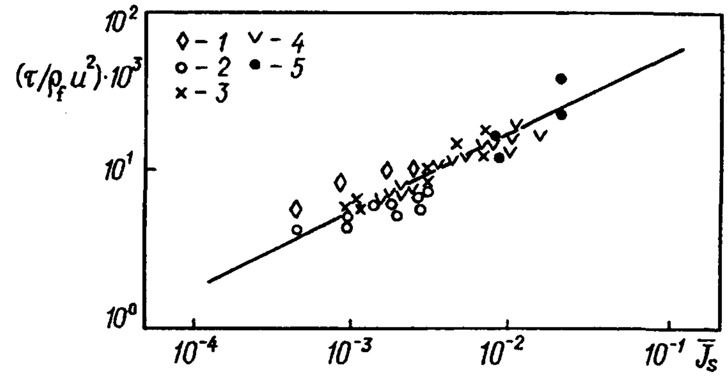

Fig. 1. Generalization of experimental data on the magnitude of the tangential stress on the riser wall: 1) $d = 0.113$ mm, 2) 0.1, 3) 0.2, 4) 1.18, 5) 0.06; 1-4) [12], 5) [13].

which agrees well with Eq. (3) (the absence of the simplex H_0/H is caused by the fact that the quantity H_0 was unchanged in the data analyzed). For a bed operating in accordance with the "chemical reactor" scheme, the following very simple equation was derived in [11] for the reactor transport zone:

$$\frac{\rho}{\rho_s} = \bar{J}_s \left(\frac{h}{H}\right)^{-0.82}, \tag{9}$$

which generalizes a large amount of literature data (10 works) concerning measurement of the local concentration of particles.

3) *Vertical pneumatic transport.* Here, Eq. (4) is valid, which because of the absence of internal circulation of particles in the system should be substantially simplified (the parameters h and H drop out of the number of governing parameters). We generalized experimental data [12, 13] on measurement of the tangential stress on the riser wall under conditions of pneumatic transport in a rarefied phase ($1 - \varepsilon \le 2 \cdot 10^{-2}$). The following equation is obtained:

$$\frac{\tau}{\rho_f u^2} = 0.17 \sqrt{\bar{J}_s}, \tag{10}$$

represented by the solid line in Fig. 1.

We can show that generally the concentration of particles in a pneumatic-transport system is calculated using the formula

$$\frac{\rho}{\rho_s} = 1 - \varepsilon = \frac{\bar{J}_s^*}{1 + \bar{J}_s^*}. \tag{11}$$

Indeed, the quantity J_s is defined by the relation

$$J_s = \rho_s (1 - \varepsilon) V = \rho V. \tag{12}$$

The velocity of motion of the particles is

$$V = \frac{u - u_t^*}{\varepsilon}. \tag{13}$$

Substitution of Eq. (13) into the formula for the mass flux (12) leads to the expression

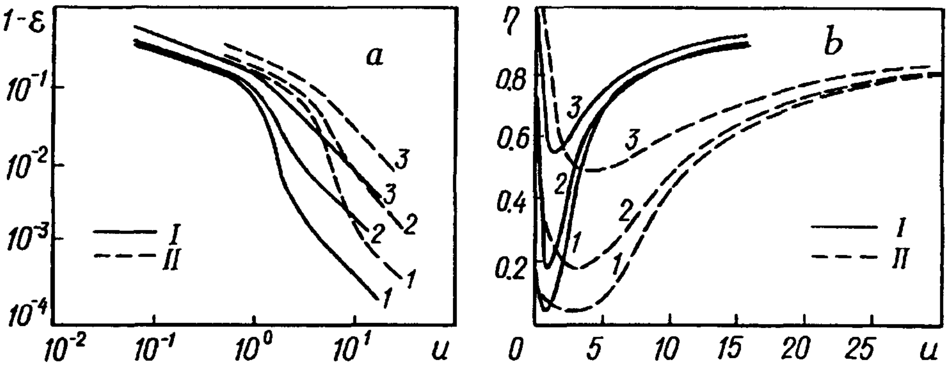

Fig. 2. Dependence of the volumetric concentration of particles (a) and the pneumatic-transport efficiency (b) on the gas filtration velocity: I) $\rho_s = 2600$ kg/m^3, $d = 0.2 \cdot 10^{-3}$ m, Ar = 770, $u_t = 1.66$ m/sec, $u_0 = 0.05$ m/sec; II) $\rho_s = 1089$ kg/m^3, $d = 1.67 \cdot 10^{-3}$ m, Ar = 184,094, $u_t = 5.9$ m/sec, $u_0 = 0.45$ m/sec; 1) $J_s = 10$ kg/(m^2·sec), 2) 50, 3) 200.

$$\frac{\rho}{\rho_s} = \frac{J_s}{\rho_s (u - u_t^*) + J_s}, \tag{11(a)}$$

which, as is easily seen, is the dimensional form of Eq. (11). Since the combination $\bar{J}_s^*$ is a function of ε ($\bar{J}_s^* = J_s/\rho_s(u - u_t^*) = J_s/\rho_s(u - u_t\varepsilon^{5.4\mathrm{Ar}^{-0.05}})$, see the Appendix), formula (11) is a transcendental equation with respect to ε. Solutions of Eq. (11) for different values of Ar, u, and J_s are shown in Fig. 2a.

For most pneumatic-transport systems $1 - \varepsilon \le 0.01$. In this case, $\bar{J}_s^* \approx \bar{J}_s \ll 1$ and Eq. (11) is simplified:

$$\frac{\rho}{\rho_s} = 1 - \varepsilon = \bar{J}_s. \tag{14}$$

From the resulting values of ε it is easy to calculate the pneumatic-transport efficiency [6]:

$$\eta = \frac{V}{u} = \frac{1}{\varepsilon} - \frac{u_t \varepsilon^{5.4\mathrm{Ar}^{-0.05}}}{\varepsilon u}. \tag{15}$$

Figure 2b illustrates calculated values of η. From a comparison of Fig. 2 a and b it is readily seen that the region of concentrations that lies between the limiting regimes (transport in dense and rarefied phases) is energetically unprofitable. As is known [6], this is the region of so-called "bubbling" regimes, which are extremely unstable.

Thus, it follows from the examples considered that relations (3) and (4) are actually a generalized form of the functional dependences of the hydrodynamic characteristics of disperse media of the indicated class on the governing dimensionless parameters. Among the latter there are the numbers Fr* and $\bar{J}_s^*$ constructed on the basis of the excess gas velocity $u - u_t^*$. This quantity is a measure of the kinetic energy of the particles, and it plays the most important role in the analysis conducted. The numbers Fr* and $\bar{J}_s^*$ and Eqs. (3) and (4), which are the rules of scaling and determine the similarities of the hydrodynamic processes, can be useful in deriving dimensionless equations for other characteristics of disperse systems with suspended particles.

Appendix

Calculation of the velocity of particle flotation under conditions of constrain

From the well-known Todes formulas for deternining u_t and u_t^* [6]

$$\mathrm{Re}_t = \frac{u_t d}{\nu_f} = \frac{\mathrm{Ar}}{18 + 0.61\sqrt{\mathrm{Ar}}} \tag{A.1}$$

and

$$\mathrm{Re}_t^* = \frac{u_t^* d}{\nu_f} = \frac{\mathrm{Ar}\, \varepsilon^{4.75}}{18 + 0.6\sqrt{\mathrm{Ar}\, \varepsilon^{4.75}}} \tag{A.2}$$

it follows that

$$\frac{u_t^*}{u_t} = \frac{(18 + 0.61\sqrt{\mathrm{Ar}})\, \varepsilon^{4.75}}{18 + 0.6\sqrt{\mathrm{Ar}\, \varepsilon^{4.75}}} . \tag{A.3}$$

From (A.3) it is possible to derive a convenient interpolational formula for calculation of u_t^*/u_t:

$$\frac{u_t^*}{u_t} = \varepsilon^{5.4\mathrm{Ar}^{-0.05}} \quad (10 \le \mathrm{Ar} \le 10^5) , \tag{A.4}$$

which is an analog of the well-known Richardson–Zaki formula [14]. When $\varepsilon = 0.4$ (a fluidized bed), $u_t^* = u_0$ ($\mathrm{Fr}_t^* = \mathrm{Fr}$) and when $\varepsilon \to 1$, $u_t^* \to u_t$ ($\mathrm{Fr}_t^* \to \mathrm{Fr}_t$, $\bar{J}_s^* \to \bar{J}_s$).

NOTATION

$\mathrm{Ar} = (gd^3/\nu_f^2)(\rho_s/\rho_f - 1)$, Archimedes number; d, diameter of the particles; D, diameter of the bed (riser); D_b, diameter of a gas bubble; $\mathrm{Fr} = (u - u_0)^2/gH$, $\mathrm{Fr}_t = (u - u_t)^2/gH$, $\mathrm{Fr}_t^* = (u - u_t^*)^2/gH$, Froude numbers; g, free-fall acceleration; h, height over the gas distributor; H, height of the bed (riser); H_0, initial height of the bed ($H_0 = M/\rho_s(1 - \varepsilon_0)S$); J_s, specific mass flux of particles; $\bar{J}_s = J_s/\rho_s(u - u_t)$, $\bar{J}_s^* = J_s/\rho_s(u - u_t^*)$, dimensionless mass fluxes of particles; M, mass of the particles in the bed (riser); Δp, head loss ($\Delta p = \rho_s(1 - \varepsilon_0)gH_0$); S, cross section of the bed (riser); u_b, absolute velocity of the bubbles; u, gas velocity; u_t^*, velocity of particle flotation under conditions of constrain; u_0, velocity at the onset of fluidization ($u_t^* \to u_0$ for $\varepsilon \to \varepsilon_0$); u_t, velocity of flotation of a single particle ($u_t^* \to u_t$ for $\varepsilon \to 1$); v, absolute particle velocity; V_b, relative velocity of the bubbles ($V_b = u_b - (u - u_0)$); ε, porosity; ε_0, porosity at the onset of fluidization; ν_f, kinematic viscosity of the gas; ρ, density; τ, tangential stress on the riser wall. Subscripts: f, gas; s, particles; t, conditions of particle flotation.

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